

Differentiability

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Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

Scan the QR code(s) or click the link for instant detailed model solutions!

Question 1

Qualification: AP Calculus AB

Areas: Limits and Continuity, Integration, Applications of Integration

Subtopics: Continuities and Discontinuities, Calculating Limits Algebraically, Average Value of a Function, Properties of Integrals, Integration Technique – Standard Functions, Differentiability

Paper: Part B-Non-Calc / Series: 2003 / Difficulty: Very Hard / Question Number: 6

6. Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5-x & \text{for } 3 < x \leq 5. \end{cases}$$

- (a) Is f continuous at $x = 3$? Explain why or why not.
- (b) Find the average value of $f(x)$ on the closed interval $0 \leq x \leq 5$.
- (c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ mx + 2 & \text{for } 3 < x \leq 5, \end{cases}$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m ?

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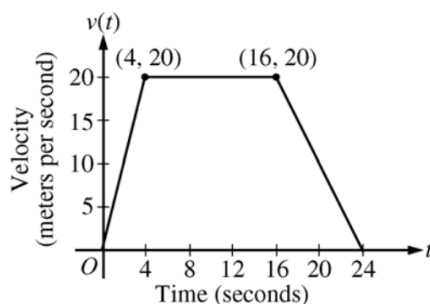
Question 2

Qualification: AP Calculus AB

Areas: Integration, Applications of Integration, Limits and Continuity, Applications of Differentiation

Subtopics: Interpreting Meaning in Applied Contexts, Kinematics (Displacement, Velocity, and Acceleration), Integration Technique – Geometric Areas, Differentiability, Derivative Graphs, Rates of Change (Average), Mean Value Theorem

Paper: Part B-Non-Calc / Series: 2005 / Difficulty: Hard / Question Number: 5



5. A car is traveling on a straight road. For $0 \leq t \leq 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.
- (a) Find $\int_0^{24} v(t) \, dt$. Using correct units, explain the meaning of $\int_0^{24} v(t) \, dt$.
- (b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.
- (c) Let $a(t)$ be the car's acceleration at time t , in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.
- (d) Find the average rate of change of v over the interval $8 \leq t \leq 20$. Does the Mean Value Theorem guarantee a value of c , for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

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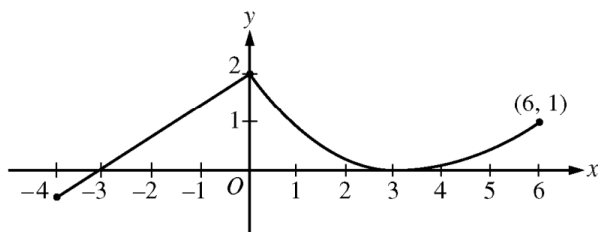
Question 3

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation

Subtopics: Differentiability, Rates of Change (Average), Mean Value Theorem, Concavity, Fundamental Theorem of Calculus (Second)

Paper: Part A-Calc / Series: 2009-Form-B / Difficulty: Somewhat Challenging / Question Number: 3



Graph of f

3. A continuous function f is defined on the closed interval $-4 \leq x \leq 6$. The graph of f consists of a line segment and a curve that is tangent to the x -axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$.
- (a) Is f differentiable at $x = 0$? Use the definition of the derivative with one-sided limits to justify your answer.
- (b) For how many values of a , $-4 \leq a < 6$, is the average rate of change of f on the interval $[a, 6]$ equal to 0? Give a reason for your answer.
- (c) Is there a value of a , $-4 \leq a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value c , $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.
- (d) The function g is defined by $g(x) = \int_0^x f(t) dt$ for $-4 \leq x \leq 6$. On what intervals contained in $[-4, 6]$ is the graph of g concave up? Explain your reasoning.

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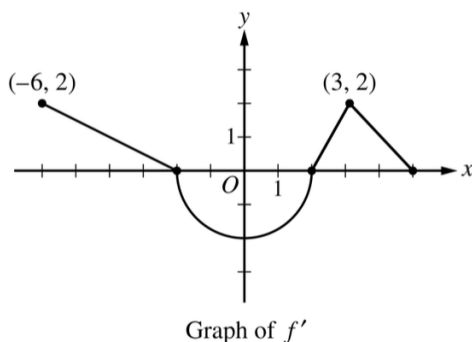
Question 4

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation

Subtopics: Integration Technique – Geometric Areas, Derivative Graphs, Increasing/Decreasing, Global or Absolute Minima and Maxima, Differentiability

Paper: Part B-Non-Calc / Series: 2017 / Difficulty: Medium / Question Number: 3



3. The function f is differentiable on the closed interval $[-6, 5]$ and satisfies $f(-2) = 7$. The graph of f' , the derivative of f , consists of a semicircle and three line segments, as shown in the figure above.
- (a) Find the values of $f(-6)$ and $f(5)$.
 - (b) On what intervals is f increasing? Justify your answer.
 - (c) Find the absolute minimum value of f on the closed interval $[-6, 5]$. Justify your answer.
 - (d) For each of $f''(-5)$ and $f''(3)$, find the value or explain why it does not exist.
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